J. Fluid Mech. (1994), vol. 266, pp. 93–119

Flow structure in the frequency-modulated wake of a cylinder

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(Received 28 September 1992 and in revised form 8 October 1993)

A cylinder is subjected to frequency-modulated (FM) excitation and the structure of its wake is characterized in terms of the modulation frequency and the frequency deviation. It is possible to destabilize or restabilize the degree of organization of the vortical structures in the near wake and thereby substantially manipulate the spectral content, relative to the case of purely sinusoidal excitation. These processes of destabilization and restabilization are attainable by varying the frequency deviation while holding the modulation frequency constant or vice versa. A phase-locked periodicity of the near-wake response is attainable at the period of the modulation frequency, as well as at double its period. This phase-locked periodicity, or lack of it, is related to the degree of organization of the wake. The structure of the far wake is strongly dependent upon the nature of the near wake modification. Either coherent or destabilized wake structure can be induced in the far wake, at a given value of nominal excitation frequency, by employing appropriate FM excitation.

1. Introduction

The flow structure from, and loading upon, an oscillating cylinder is of central importance in the area of flow-induced vibrations. However, there are a number of other important applications involving steady flow past an oscillating cylinder, pulsating flow past a stationary cylinder, or a combination of these possibilities. The phenomena of noise generation, mixing, heat transfer and combustion are related to the nature of the wake from the cylinder. In many applications, a principal objective is to control the spectral content of the wake. It may be desirable, for example, to transform noise generation from a spectrum having resonant peaks to a broadband spectrum, to enhance the effectiveness of mixing at low frequencies accompanied by manipulation of the spectral content, or to modify the convective heat transfer of bodies immersed in wakes of other bodies through alteration of the spectral content of the flow incident upon each of the bodies.

Sinusoidal excitation of the cylinder generates a number of classes of response of the near wake, ranging from locked-in vortex formation to chaotic fluctuations. A summary of recent investigations for this type of forcing is given by Nakano & Rockwell (1993) in a companion paper to the present one. Therein, the consequence of amplitude-modulated (AM) excitation is considered and its effectiveness in inducing low-order chaotic response is addressed. In the present work, we explore the consequences of frequency-modulated (FM) excitation, for which the frequency changes continuously with time at constant amplitude. Whereas the focus of the AM

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study was on the possible routes to chaos, the case of FM excitation addressed here focuses on the nature and extent of the region of locked-in wake response. Indeed, it is possible to attain very substantial changes of the form and extent of the locked-in response by proper selection of the modulation frequency and frequency deviation of the FM forcing.

Even in the absence of excitation, the wake from a stationary cylinder exhibits a global instability that embodies a region of absolute instability. The properties of these globally unstable wakes and the relevant literature are described by Huerre & Monkewitz (1990). A principal characteristic of these wakes is highly organized, self-sustained vortex formation. Sinusoidal excitation at a frequency sufficiently far from that of the inherent global instability can induce low-order chaotic modulations of the wake, as described most recently by Karniadakis & Triantafyllou (1989*a*, *b*) and the works described therein including those of Sreenivasan (1985), Van Atta & Gharib (1987), and Olinger & Sreenivasan (1988).

An issue of major importance is the possibility of disrupting the limit-cycle oscillations of the near wake, which arise from the global instability, by FM excitation. Furthermore, in situations where non-sinusoidal oscillations occur from outside the lock-in range, the possibility of regaining a locked-in state via FM excitation is an intriguing prospect. One expects such FM excitation to induce unique classes of vortex arrangements in the near-wake region immediately downstream of the cylinder. The nominal excitation frequency, the frequency deviation from this nominal value, and the modulation frequency are all expected to exert a substantial influence on the near-wake vortex pattern. As the wake evolves in the downstream direction, the unstable arrangement of vortices in the near wake may lead to the rapid onset of large-scale, incoherent vortical structures in the intermediate- and the far-wake regions. On the other hand, if the near-wake vortex arrangement induced by FM excitation is locked-in to the cylinder motion, its imprint may persist well into the far wake. These structural changes of the wake will be reflected, of course, in the spectral content. Either sharp spectral peaks or broadband spectra may be attainable, depending upon the type of FM excitation.

This investigation attempts to define these central features of the frequency-modulated wake by global flow visualization of the near- and far-wake regions and corresponding representations of the unsteady velocity fluctuations in terms of spectra. Such a complementary approach allows both instantaneous and time-averaged insight.

Section 2 describes the experimental system and techniques and §3 gives an overview of the classes of wake response. Section 4 introduces the concept of expansion of the region of locked-in response, relative to that occurring for purely sinusoidal excitation. Section 5 addresses techniques of destabilizing the wake by either variations in modulation frequency or frequency deviation; on the other hand, §6 reveals the possibility of restabilizing the wake to a periodic state from a non-deterministic state by appropriate excitation. Finally, an overall assessment of these observations is given in §7.

2. Experimental system and techniques

A free-surface water channel was employed for the investigations described herein. It had a cross-section of 610 × 610 mm. The cylinder was mounted in such a manner that the excitation and support mechanisms were above the free surface, allowing an unobstructed flow past the cylinder, which had a length $L = 520$ mm and a diameter $D = 4.76$ mm, giving a length to diameter ratio of $L/D = 109$. The free-stream velocity was $U = 34.4\ \text{mm s}^{-1}$, corresponding to a Reynolds number based on cylinder diameter of $Re = 136$. For these conditions, the inherent vortex shedding frequency $f_0^* = 1.367$ Hz.

The cylinder was forced with a high-resolution Daedel traverse table, which was driven by a high-resolution Compumotor. This motor operates at about 10000 steps per revolution, providing a smooth output equivalent to a DC servomotor. It was driven by the laboratory microcomputer, which was programmed to provide the desired frequency-modulated forcing functions.

The flow visualization was accomplished using the hydrogen-bubble technique. A 0.025 mm diameter platinum wire mounted in the horizontal plane at a distance of 10 mm upstream of the cylinder allowed close-up visualization of the near-wake vortical structures. In order to examine the evolution of the wake in the downstream direction, this wire was moved to locations corresponding to $x/D = 5$, 20 and 35, where $x = 0$ corresponds to the centre of the cylinder. The images of the hydrogen-bubble timelines were reflected from a mirror located below the bottom of the test section of the water channel and oriented at an angle of 45°. The reflected image was then projected into a video camera, and in turn recorded on a high-speed video system at 120 frames per second. Further details of this technique of flow visualization are given by Takmaz & Rockwell (1993).

Measurements of the velocity field in the wake were accomplished using a laser-Doppler anemometer system involving a 2 W Argon-ion laser, the usual optical train, and a beam expander, allowing acquisition of data in the backscatter mode. The laser signal was processed on the laboratory microcomputer, and the usual precautions were followed to account for aliasing and proper selection of sampling time. The LDA signal was filtered at 0.1 and 6 Hz. Considerable effort was devoted to identification of the most physically significant locations for acquiring velocity spectra, accounting for the maximum displacement amplitude Y_e from the equilibrium position of the cylinder. They are: $x/D = 5$ and $y/D = Y_e/D + 1.8$, corresponding to the near wake and $x/D = 40$ and $y/D = 3$, representing the downstream region of the wake, termed herein the far wake. These locations at the outer edge of the wake allowed identification of the predominant spectral components arising from the large-scale structures and their interaction.

3. Overview

The frequency-modulated (FM) excitation of the cylinder displacement is according to

$$y(t) = -Y_e \sin [2\pi f_e t + \{(\Delta f_e/f_e)/(f_m/f_e)\} \sin 2\pi f_m t],$$

in which f_e is the nominal (carrier) frequency, Δf_e is the frequency deviation, and f_m is the modulation frequency. Herein, we normalize each of these frequency components as f_e/f_0^*, $\Delta f_e/f_e$, and f_m/f_e, in which f_0^* is the inherent vortex formation frequency from the stationary cylinder.

In describing the response of the wake structure, we employ the following definitions. *Lock-in at frequency* f_e corresponds to the shedding of one vortex pair per f_e cycle. That is, two successive vortices of opposite sense are shed during each f_e cycle of the cylinder motion, as indicated in figure 1 (*a*). The circles of solid lines and dotted lines represent respectively the instants at which counterclockwise and clockwise vortices are formed in the near wake. It is evident that vortices are formed at approximately the same Y-location of the cylinder from one cycle to the next. In the case of purely sinusoidal excitation, this condition for lock-in is satisfied exactly. *Non-*

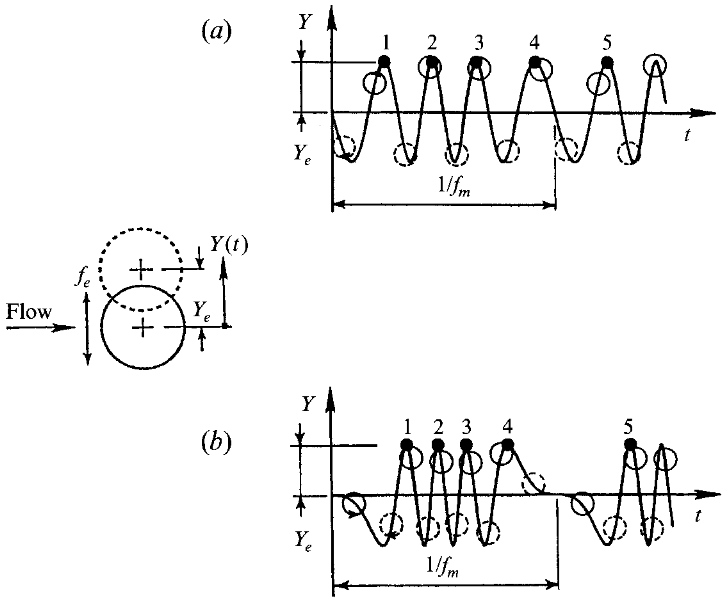


FIGURE 1. Schematic illustrating (*a*) locked-in response at excitation frequency f_e and periodic response at modulation frequency f_m; and (*b*) non-locked-in response at excitation frequency f_e and periodic response at modulation frequency f_m.

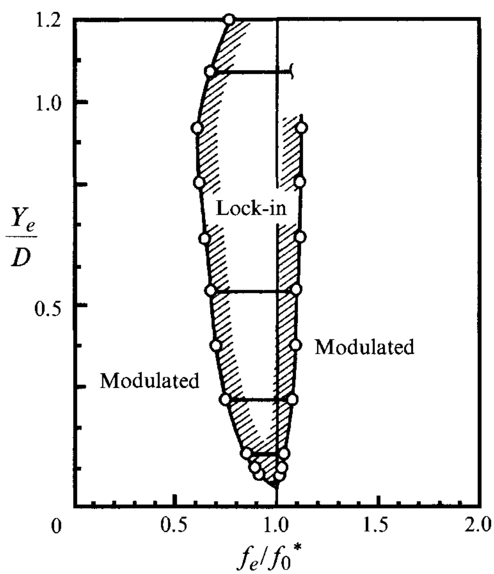


FIGURE 2. Illustration of extent of region of locked-in vortex formation as a function of oscillation amplitude Y_e/D of cylinder for purely sinusoidal excitation at frequency f_e relative to natural vortex formation frequency f_0^*.

lock-in at frequency f_e occurs when more than two vortices are formed during each f_e cycle; it is represented in figure 1 (*b*). Such non-lock-in typically occurs when the period of the cylinder displacement is much longer than that of the natural vortex formation. Irrespective of whether locked-in or non-locked-in vortex formation occurs, the overall pattern of vortex formation can be f_m-*periodic*, meaning that it repeats at a frequency f_m, or period $1/f_m$, as shown in figure 1 (*a*, *b*). A less-ordered version of this periodicity

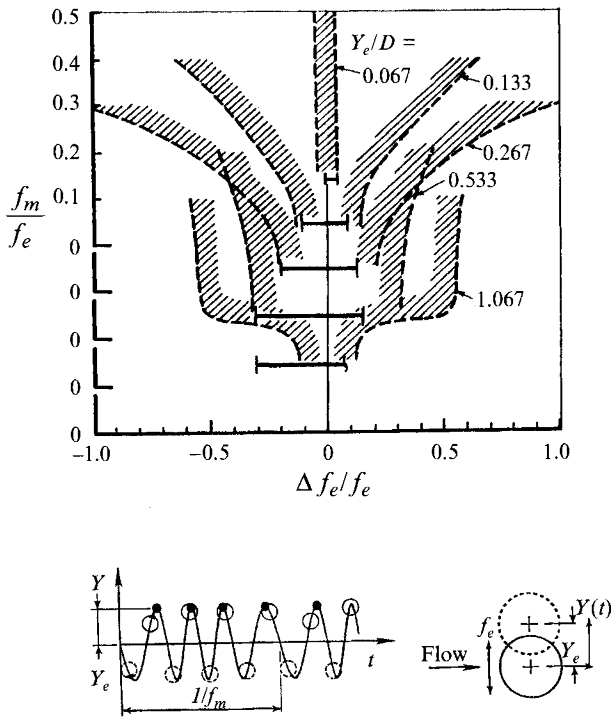


FIGURE 3. Definition of regions of locked-in response at excitation frequency f_e on plane of dimensionless modulation frequency f_m/f_e versus frequency deviation $\Delta f_e/f_e$. At each value of dimensionless amplitude Y_e/D of cylinder oscillation, region of locked-in response occurs within and above hatched region. Bold horizontal lines represent extent of locked-in response to purely sinusoidal excitation taken from figure 2. Nominal excitation frequency $f_e/f_0^* = 0.95$.

at f_m can occur at a frequency $\frac{1}{2}f_m$, i.e. it extends over $2f_m$ cycles; it is referred to a $2f_m$-*periodic*. It is possible to induce states of increasing disorder of the near-wake vortex pattern by varying the frequency deviation $\Delta f_e/f_e$ and modulation frequency f_m/f_e. These states are defined in terms of the degree of lock-in at f_e and periodicity at f_m.

In order to provide a reference case for describing the response to FM excitation, we first address the concept of lock-in for sinusoidal excitation, illustrated in figure 2. In accord with the foregoing definition, it corresponds not only to the formation of one vortex pair during each f_e cycle, but also to the shedding of each vortex at the same instantaneous displacement $Y(t)$ during each cycle. As defined by the hatched lines in figure 2, the region of lock-in changes width with increasing amplitude. The bold horizontal bars designate the reference amplitudes selected for FM excitation; they are superposed on the FM response diagram.

An overview of the response of the near wake to FM excitation is given in figure 3 on a plane of f_m/f_e *vs.* $\Delta f_e/f_e$. This response diagram was acquired for slight detuning of the forcing frequency from the inherent instability frequency of vortex formation from the stationary cylinder, namely, $f_e/f_0^* = 0.95$. The regions of locked-in vortex formation from the sinusoidally excited cylinder are indicated by the bold horizontal lines at each value of amplitude Y_e/D; they correspond to those of figure 2. The value of frequency ratio $\Delta f_e/f_e = 0$ on the horizontal axis of figure 3 corresponds to the case of sinusoidal excitation at $f_e/f_0^* = 0.95$. For the case of FM excitation, the regions of locked-in vortex formation having periodicity at frequency f_m lie in the regions interior to and above the bold dashed lines; these regions are indicated by hatched lines. That

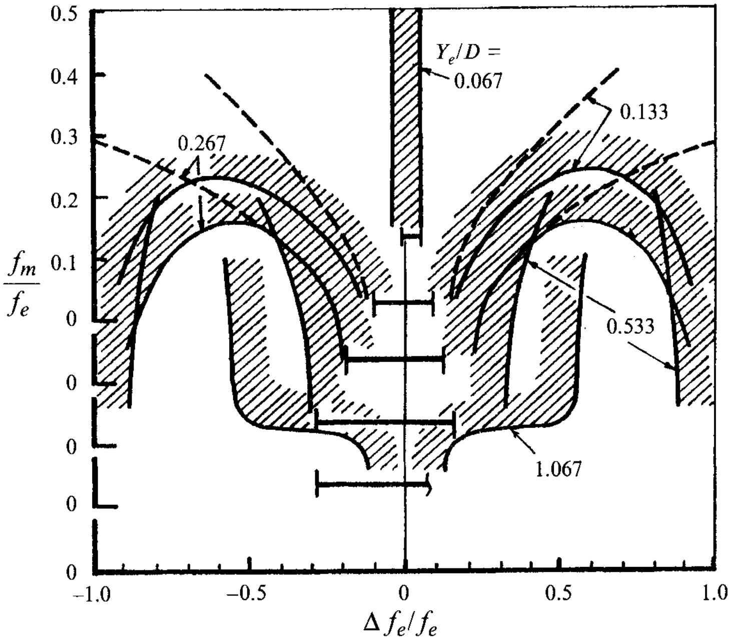


FIGURE 4. Domains of periodic response at modulation frequency f_m, defined by regions within and above hatched regions. Locked-in response at excitation frequency f_e does not necessarily occur in these f_m-periodic domains. For comparison, dashed lines represent boundaries of f_e-lock-in response, taken from figure 3. Bold horizontal lines represent the extent of locked-in response for purely sinusoidal excitation, taken from figure 2. $f_e/f_0^* = 0.95$.

is, for FM excitation within the indicated ranges of $\Delta f_e/f_e$, there is locked-in vortex formation, i.e. formation of one pair of vortices during each cycle of the cylinder displacement and repetition of the overall vortex pattern at a frequency f_m. It is evident that FM excitation provides the possibility of substantially widening the region of phase-locked vortex formation beyond that attainable with classical sinusoidal excitation. In fact, at a value of excitation amplitude $Y_e/D = 0.267$, the so-called region of lock-in extends over the entire range of frequency deviation $\Delta f_e/f_e$ up to 1.0.

If we lift the foregoing restriction that a vortex pair is formed during each and every f_e cycle of the cylinder, then locked-in vortex formation does not occur over each f_e cycle, but it is still possible to attain patterns of vortex formation that are repetitive at frequency f_m, i.e. f_m-periodic. Figures 1 (*a*) and 1 (*b*) show respectively cases of: locked-in, f_m-periodic; and non-f_e-lock-in, f_m-periodic. The circles of solid lines and dashed lines represent respectively the instants at which counterclockwise and clockwise vortices are fully formed in the near wake. As suggested in figure 1 (*b*), during those portions of the FM excitation for which there are long time delays, i.e. between peaks 4 and 5, a number of vortices of unlike sense may be formed. In the response diagram of figure 4, the bold solid lines and associated hatched markings indicate the regions of f_m-periodic response, without consideration of whether f_e-lock-in occurs. The bold dashed-lines correspond to the f_e-lock-in, f_m-periodic boundaries of figure 3. At the largest amplitude $Y_e/D = 1.067$, the bold continuous lines are coincident with the dashed lines of figure 3, meaning that the condition of f_m-periodicity with f_e cycle lock-in is maintained. For the amplitude $Y_e/D = 0.533$, the same observation applies at lower values of $\Delta f_e/f_e$; however, a second branch of the $Y_e/D = 0.533$ curve appears at high values of $\Delta f_e/f_e$ representing f_m-periodic response without f_e-lock-in. At the lower values of amplitude $Y_e/D = 0.133$ and 0.267, the f_m-periodic response curves closely follow the corresponding f_e-lock-in curves of figure 3 at small values of frequency deviation $\Delta f_e/f_e$, but depart substantially at larger values of $\Delta f_e/f_e$. We therefore see that for excitation amplitudes in the range $0.133 \leqslant Y_e/D \leqslant 0.533$, at a given value of f_m/f_e, the wake undergoes a series of transitions as $\Delta f_e/f_e$ is increased: f_m-periodic (in most cases with f_e-lock-in); non-f_m-periodic; and f_m-periodic without f_e-lock-in. However, such a transition between modes of response of the wake can be precluded by forcing at sufficiently high values of modulation frequency f_m/f_e, where f_m-periodic response is attainable over the entire range of frequency deviation $\Delta f_e/f_e$.

Extensive study of the wake response, using flow visualization, as well as velocity measurements in terms of time traces and spectra, reveals that there are several basic classes of response of the near- and far-wake structure that involve: enhancement of the extent of locked-in response; destabilization of the locked-in response; and restabilization of the non-locked-in wake structure. In the following, these basic types of response are described in detail.

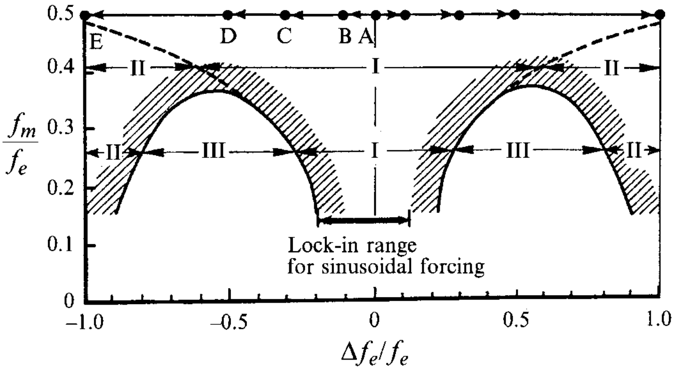
4. Enhancement of coherence of wake response: expansion of lock-in range

As indicated in figure 3, application of FM excitation can generate f_m-periodic, f_e locked-in response of the wake structure, extending over large values of frequency deviation. The schematic of figure 5 illustrates this type of response. The region above the solid curve represents f_m-periodic with non-f_e-lock-in response; the region above the dashed line corresponds to f_m-periodic with f_e lock-in response. The bold horizontal line with arrows represents the maximum widening of the range of f_e lock-in response, which is addressed here. At this relatively high value of modulation frequency $f_m/f_e = 0.5$, the flow structure is locked-in over the entire range of frequency deviation $\Delta f_e/f_e$, i.e. $0.1 \leqslant \Delta f_e/f_e \leqslant 1.0$.

The photos of figure 6 illustrate the vortex patterns at the extreme values of $\Delta f_e/f_e = 0.1$ and 1.0, corresponding to the operating points B and E in figure 5. Note that the flow structure corresponding to pairs of photos 1, 3 and 2, 4 in figure 6 is highly repeatable, corresponding to f_m-periodic response with lock-in at frequency f_e.

The spectra of the near-wake velocity fluctuation taken at $x/D = 5$, are shown in the left column of figure 7 for values of frequency deviation $\Delta f_e/f_e = 0.1$, 0.3, 0.5, and 1.0, as well as for the value of frequency deviation $\Delta f_e/f_e = 0$, corresponding to the case of purely sinusoidal excitation. The resonant peak at the nominal excitation frequency f_e (indicated by the dot), which is characteristic of purely sinusoidal excitation, is maintained over the entire range of $\Delta f_e/f_e$. In addition, however, there appear well-defined sidebands at $f_e \pm f_m$. The amplitudes of these sidebands, relative to the nominal, or carrier, frequency f_e increase with increasing frequency deviation. Spectra taken in the far wake† at $x/D = 40$ are given in the right column of figure 7. Whereas the case of purely sinusoidal excitation shows only a small resonant peak at the excitation frequency, the FM wake shows dominance of the component at the modulation frequency f_m. In fact, for higher values of frequency deviation $\Delta f_e/f_e$, the peak at f_e disappears within the background turbulence level. The vortex interactions in the downstream wake therefore filter out completely the upper f_m sideband and produce a very large amplitude of the lower sideband.

† By 'far-wake' of the oscillating cylinder is meant a distance of at least ten vortex wavelengths downstream of the cylinder. This definition contrasts with the classical one for wakes from bodies for which the asymptotic states of the mean and fluctuating velocity fields are of interest.



I ≡ Periodic at f_m; locked-in at f_e,
II ≡ Periodic at f_m; non-locked-in at f_e,
III ≡ Non-Periodic at f_m; non-locked-in at f_e.

FIGURE 5. Excitation conditions (designated by dots) for flow visualization corresponding to expansion of region of locked-in response on plane of dimensionless modulation frequency f_m/f_e *vs.* frequency deviation $\Delta f_e/f_e$. $f_e/f_0^* = 0.95$; $Y_e/D = 0.267$.

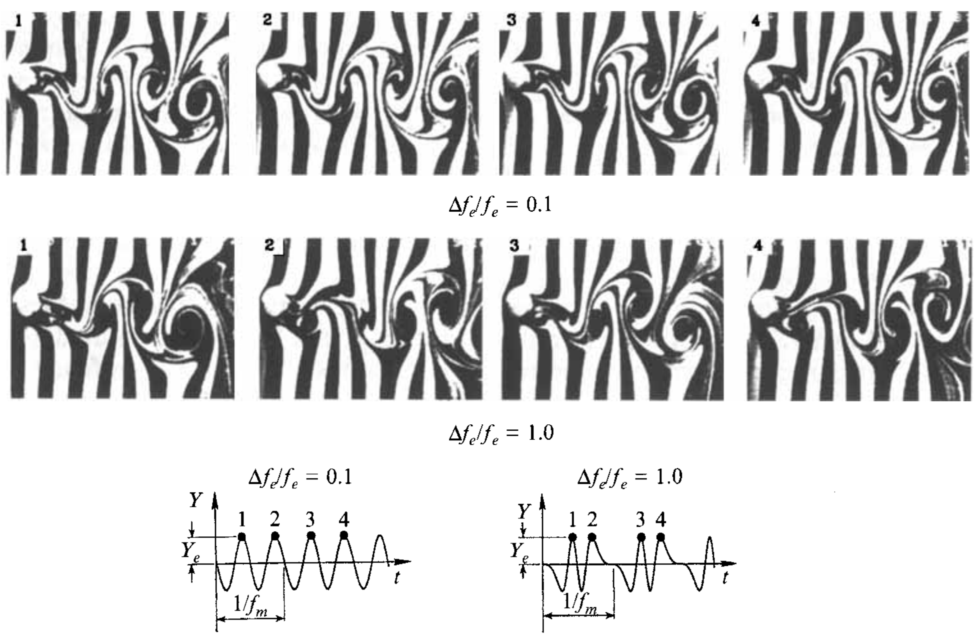


FIGURE 6. Flow structure in near-wake region corresponding to relatively small and large values of frequency deviation $\Delta f_e/f_e = 0.1$ and 1.0. $f_m/f_e = \frac{1}{2}$; $Y_e/D = 0.267$; $f_e/f_0^* = 0.95$.

Flow visualization in downstream regions of the wake is given in figure 8. The photos therein correspond to the extreme values of frequency deviation $\Delta f_e/f_e = 0.1$ and 1.0. In these visualization series, the bubble wire was located at successive positions $x/D = 5$, 20 and 35. The streamwise extent of each photograph is $\Delta x/D = 20$. At each wire location x/D, two successive visualization photos are illustrated; they have a time spacing of $\Delta t = 1/f_m$, in order to illustrate the degree of phase locking of the flow structure at successively larger streamwise distances.

At the lowest value of frequency deviation, $\Delta f_e/f_e = 0.1$, the structure of the wake is

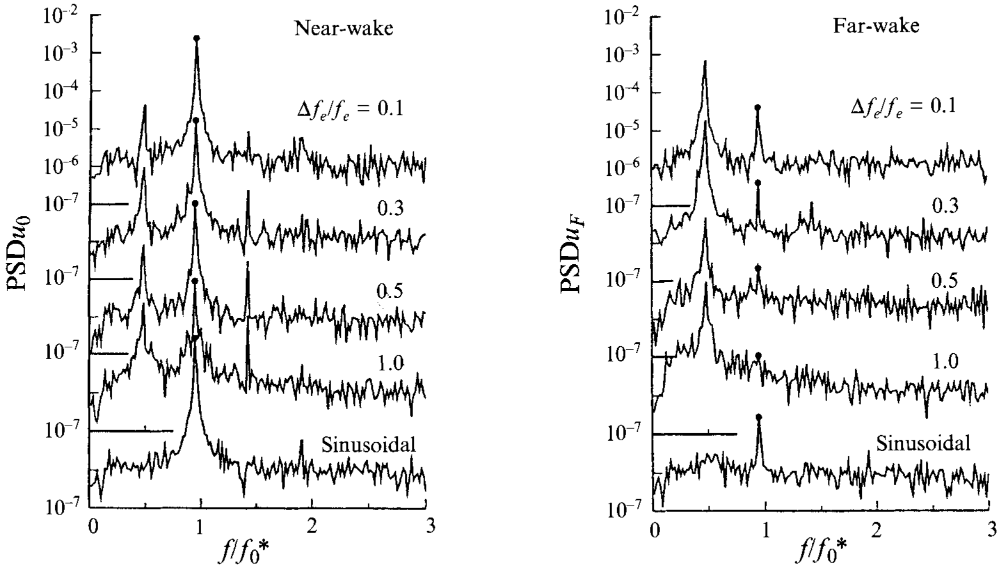


FIGURE 7. Spectra in near- and far-wake regions corresponding to excitation conditions that generate expanded lock-in region as function of frequency deviation $\Delta f_e/f_e$. $f_m/f_e = \frac{1}{2}$; $f_e/f_0^* = 0.95$; $Y_e/D = 0.267$.

essentially identical at $t = 0$ and $t = 1/f_m$ at all downstream locations; it is therefore phase locked over the entire streamwise extent of the wake. At $x/D = 5$ and 20, the basic form of the vortex street of the FM wake approximates that of the sinusoidally excited wake. At the largest streamwise distance, represented by $x/D = 35$, however, the FM wake shows the predominance of larger-scale vortical structures interspersed among smaller ones; this structure represents the onset of predominance of the f_m component, leading to the spectral content given in figure 7. Particularly remarkable is the fact that this small frequency deviation $\Delta f_e/f_e = 0.1$ has no significant effect on the wake structure for a distance of at least $x/D = 20$. Apparently enough residual energy at the modulation frequency is retained along the streamwise extent of the wake to allow emergence of this spectral component at large x/D through vortex–vortex interactions.

Visualization at the largest value of frequency deviation $\Delta f_e/f_e = 1.0$ in figure 8 shows an essentially phase-locked response of the wake at the modulation period $1/f_m$ for $x/D = 5$, 20, and 35. Even at locations immediately downstream of the near-wake region, i.e. $x/D = 5$, there is substantial deviation of the wake structure from that of the purely sinusoidal case. At $x/D = 20$ and 35, the FM wake is dominated by larger-scale vortical structures. This early predominance of the large-scale structures is compatible with the complete dominance of the f_m component, lack of a significant f_e component, and general increase in the broadband background level of the spectrum at $\Delta f_e/f_e = 1.0$ in figure 7.

The effect of lower values of modulation frequency f_m/f_e and higher values of amplitude Y_e/D on the boundaries of locked-in and periodic response are indicated in figures 3 and 4. A representative response of the near wake for these extreme conditions is shown in figure 9. It is evident that drastic changes in the vortex structure are attainable, relative to the higher f_m/f_e and lower Y_e/D of figure 6 and the case of purely sinusoidal forcing also shown in figure 9. Yet, the f_m-periodicity of the wake structure is maintained. Examination of corresponding visualization of the far wake (not shown)

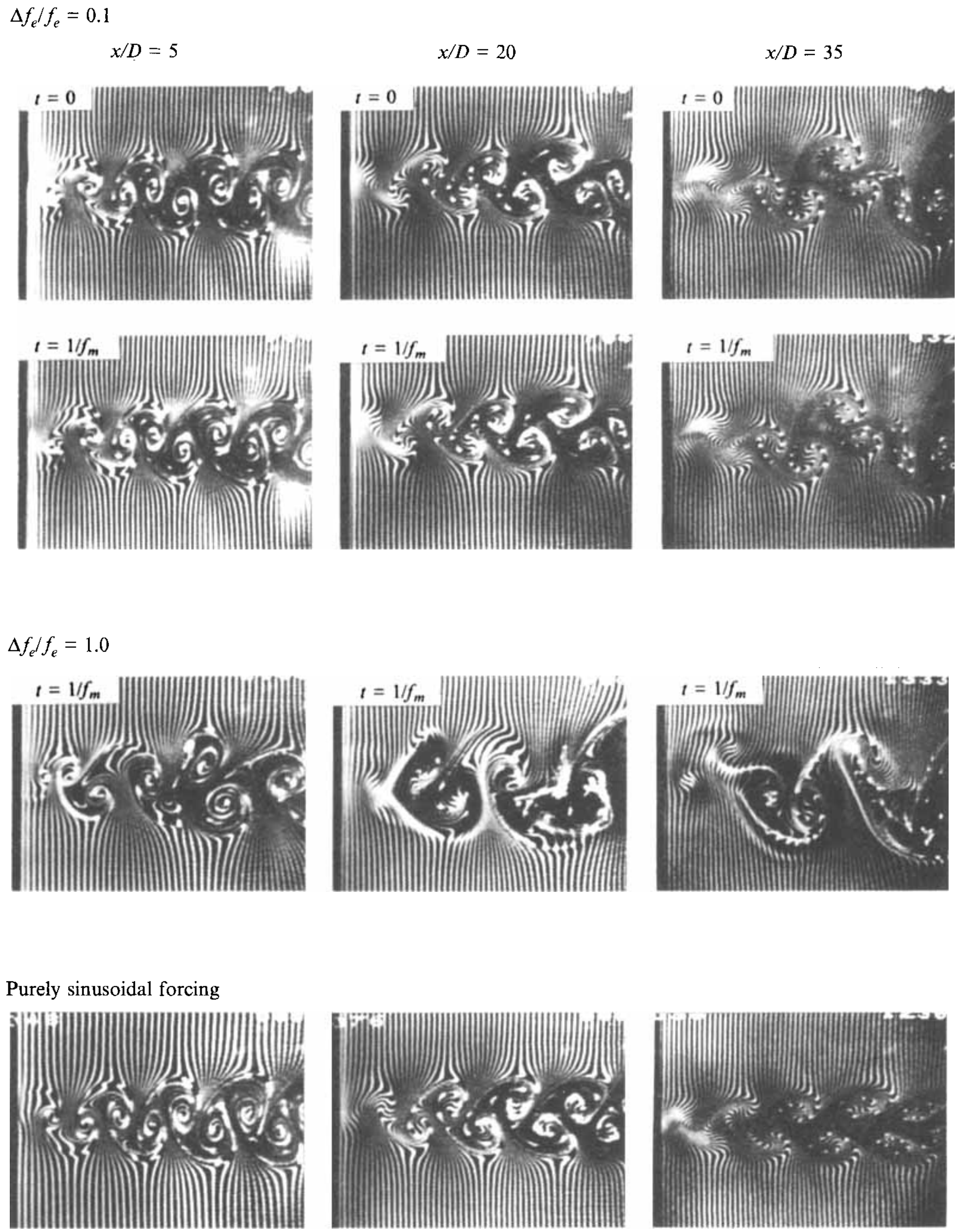


FIGURE 8. Streamwise evolution of vortex patterns corresponding to excitation conditions that generate expansion of locked-in region of wake response at relatively small and large frequency deviations $\Delta f_e/f_0 = 0.1$ and 1.0. $f_m/f_e = \frac{1}{2}$; $f_e/f_0^* = 0.95$; $Y_e/D = 0.267$.

reveals that this periodicity at f_m extends into that region. The corresponding spectra, however, show important distinctions relative to those of figure 7. In the near-wake, the lower-frequency sidebands at $f_m = \frac{1}{4}f_e$, $\frac{1}{2}f_e$ and $\frac{3}{4}f_e$ are commensurate with, or even exceed, the carrier frequency f_e. All peaks are sharp, however, corresponding to the f_m-periodic response. There are no significant peaks above f_e, except for the first sideband

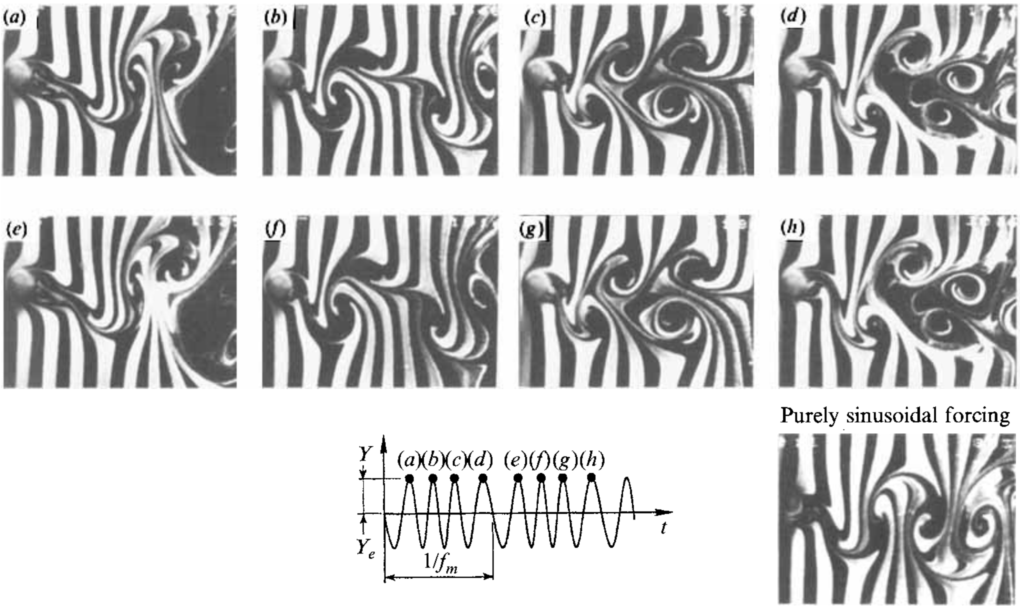


FIGURE 9. Near-wake structure corresponding to periodic response at modulation frequency f_m and locked-in response at excitation frequency f_e for values of frequency deviation $\Delta f_e/f_e = 0.3$. $f_m/f_e = \frac{1}{4}$; $f_e/f_0^* = 0.95$; $Y_e/D = 0.533$.

at $f_e + \frac{1}{4}f_e$; this means that the near wake has effectively attenuated or filtered out the f_m sidebands above the carrier frequency f_e (corresponding to $f/f_0^* = 0.95$). In other words, the process of FM excitation redistributes the fluctuation energy over the range of frequencies lower than the carrier frequency f_e. The spectrum of the far wake exhibits a dominant peak at the lowest sideband component; in effect, all other discrete components are filtered out.

5. Transition from lock-in to destabilization

5.1. *Destabilization by variation of modulation frequency*

Transition of the near-wake structure from a locked-in response, corresponding to the f_m-periodic, f_e lock-in case, to complete destabilization with no apparent periodicity, can be achieved by variation of the modulation frequency f_m/f_e at a given value of frequency deviation $\Delta f_e/f_e$. To illustrate this concept, we consider successively smaller values of modulation frequency f_m/f_e represented by the symbols B–E in figure 10. These symbols correspond respectively to the values of modulation frequency $f_m/f_e = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$. At the modulation frequency $f_m/f_e = \frac{1}{2}$, the flow structure (not shown) is f_m-periodic and locked-in at f_e. A decrease in the value of modulation frequency to $f_m/f_e = \frac{1}{3}$ produces a doubling of the period of the wake repetition, as illustrated in figure 11. The wake structure is highly repetitive every other f_m cycle, evidenced by the similarity of the following pairs of photos: 1, 7; 2, 8; 3, 9; and so on. Selected features of this $2f_m$-periodic response were preliminarily described by Nakano & Rockwell (1991 *b*). This $2f_m$-periodic response is analogous to the period doubling associated with, for example, the onset of chaotic response in closed systems subjected to purely sinusoidal oscillation including the Rayleigh–Bénard instability investigated by Gollub & Benson (1980), the self-excited secondary instability from a uniform circular cylinder revealed by Karniadakis & Triantafyllou (1990, 1991) and Tombonlides, Triantafyllou

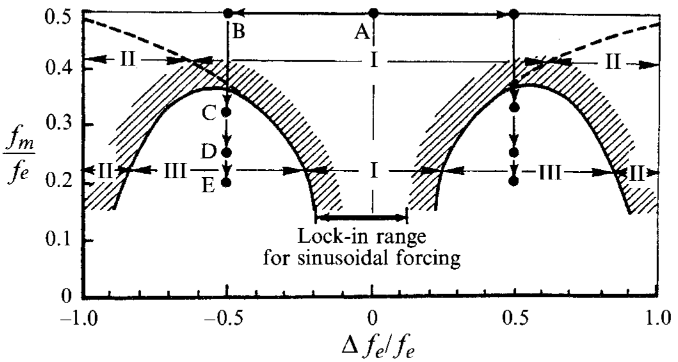


FIGURE 10. Excitation conditions (indicated by dots) for flow visualization representing states of increasing disorder during process of destabilization of wake structure on plane of dimensionless modulation frequency f_m/f_e versus frequency deviation $\Delta f_e/f_e$. $f_e/f_0^* = 0.95$; $Y_e/D = 0.267$.

& Karniadakis (1992), and by the forced instability from a non-uniform cylinder described by Rockwell, Nuzzi & Magness (1990, 1991) and Nuzzi, Magness & Rockwell (1992). At the lowest value of modulation frequency $f_m/f_e = \frac{1}{5}$ (not shown), the wake loses its periodicity, with no indication of either f_m- or $2f_m$-periodic response.

Spectra taken in the near wake are illustrated in figure 12(*a*) for values of modulation frequency $f_m/f_e = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$. For the near wake, at $f_m/f_e = \frac{1}{2}$, the wake exhibits a spectrum having the form of a typical locked-in response at the nominal forcing (carrier) frequency f_e, and sharp spectral peaks at the two sideband frequencies f_m. The onset of $2f_m$-periodic response at the modulation frequency $f_m/f_e = \frac{1}{3}$ produces a large number of spectral peaks. Both below and above the nominal excitation frequency f_e there occur peaks corresponding to $f_e + \frac{1}{6}f_e$, $\frac{2}{6}f_e$, etc. representing the $2f_m$-periodic response. Further lowering of the modulation frequency to a value $f_m/f_e = \frac{1}{4}$ produces substantial broadening of the spectral peaks at not only the nominal excitation frequency f_e, but also at its lower sidebands, represented by peaks at $f_e - \frac{1}{4}f_e$, through $\frac{3}{4}f_e$. Finally, at the lowest value of modulation frequency $f_m/f_e = \frac{1}{5}$, there is complete loss of periodicity in the spectrum of the wake response.

The consequence of this decreasing coherence of the near wake on the far wake is illustrated in figure 12(*b*). At the highest modulation frequency $f_m/f_e = \frac{1}{2}$, the f_m component dominates the spectrum. At $f_m/f_e = \frac{1}{3}$, discernible spectral components are detectable at $nf_e/6$, corresponding to the lower sidebands of the spectrum of figure 12(*a*). At the lowest values of modulation frequency $f_m/f_e = \frac{1}{4}$ and $\frac{1}{5}$, the spectral broadening is evident, with one, or at most two, discernible f_m sidebands protruding above the broadband response. Comparing these spectra with that of purely sinusoidal excitation, it is evident that, at sufficiently low values of modulation frequency f_m/f_e, there is a substantial increase in the level of the fluctuations at low frequency. The amplitude of the power spectral density is increased by a factor of over one-and-a-half orders of magnitude. This increase in amplitude at the low frequencies is evident in relatively broadband form at modulation frequencies $f_m/f_e = \frac{1}{4}$ and $\frac{1}{5}$ and in a more organized form at $f_m/f_e = \frac{1}{3}$. Moreover, we note that the low-frequency peak at f_m for a modulation frequency $f_m/f_e = \frac{1}{2}$ extends nearly two-and-a-half orders of magnitude above the background level corresponding to purely sinusoidal excitation.

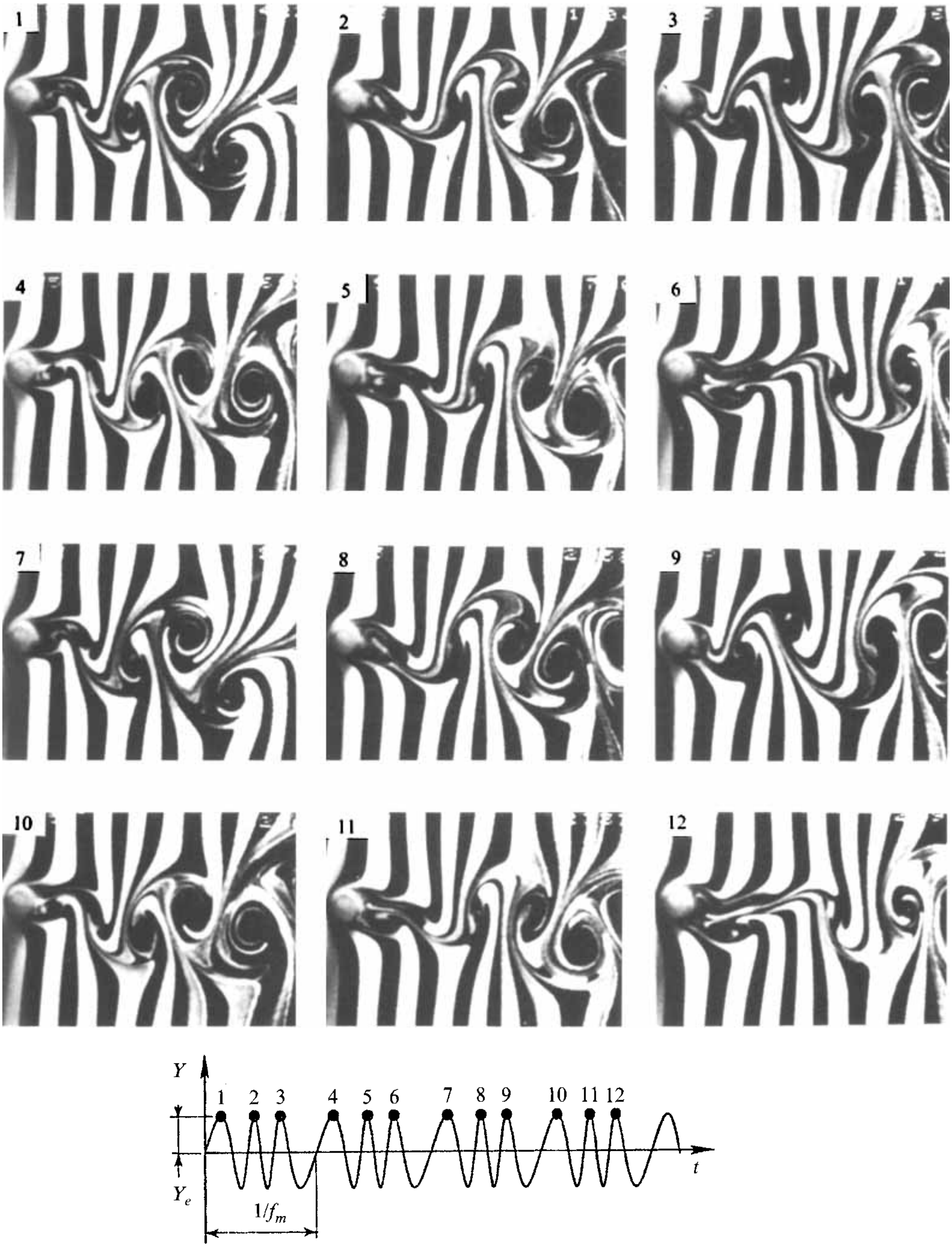


FIGURE 11. Near-wake structure corresponding to period-doubled response, i.e. periodic at half the modulation frequency f_m and non-locked-in response at excitation frequency f_e. $f_m/f_e = \frac{1}{3}$; $\Delta f_e/f_e = 0.5$; $f_e/f_0^* = 0.95$; $Y_e/D = 0.267$.

If the nominal excitation frequency is retuned to its matched condition at $f_e/f_0^* = 1$ from the condition $f_e/f_0^* = 0.95$, the near-wake spectral response takes the form shown in figure 12(c). In general, the discrete peaks of the spectra are more pronounced, relative to the broadband background, than those corresponding to

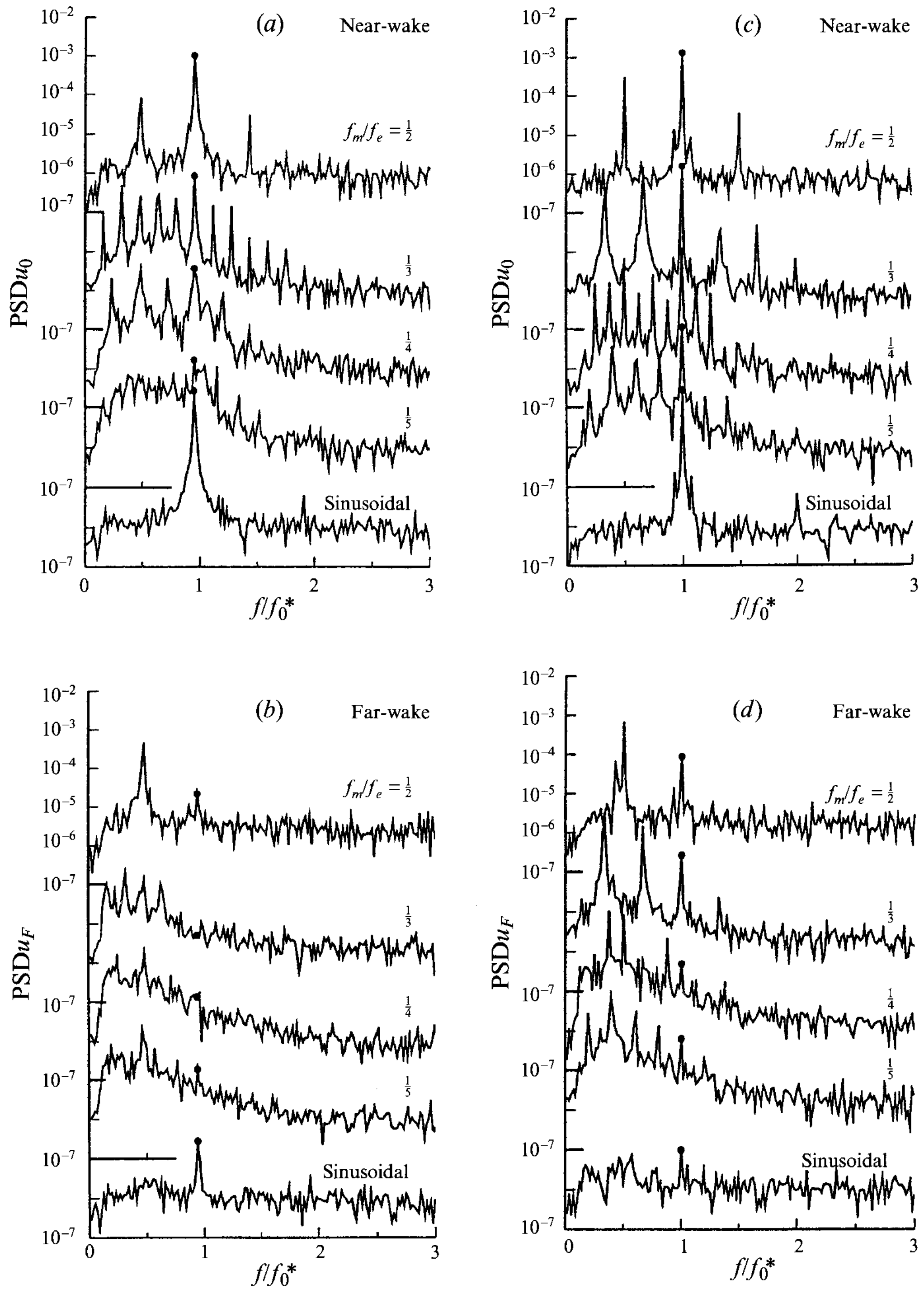


FIGURE 12. Spectra in near-wake and far-wake regions corresponding to states of increasing disorder associated with destabilization process, represented by various values of dimensionless modulation frequency f_m/f_e. $\Delta f_e/f_e = 0.5$; $f_e/f_0^* = 0.95$ (a, b) and 1.00 (c, d); $Y_e/D = 0.267$.

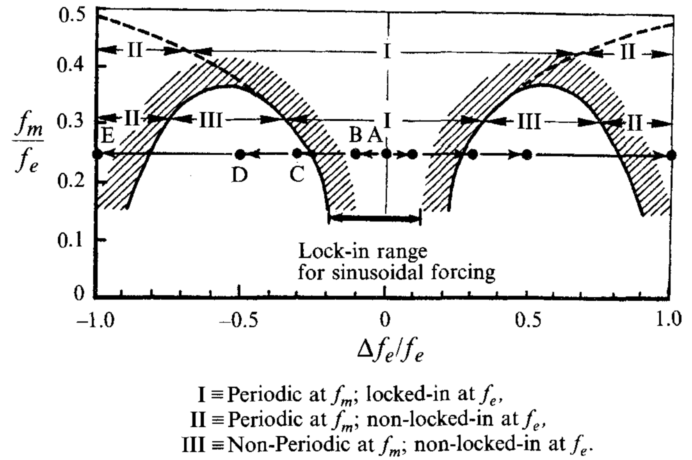


FIGURE 13. Excitation conditions (designated by dots) for flow visualization corresponding to increasing states of disorder attained by increasing frequency deviation $\Delta f_e/f_e$. In the limit of large $\Delta f_e/f_e$, stable, locked-in wake response is recovered. These operating conditions are illustrated on plane of dimensionless modulation frequency f_m/f_e versus frequency deviation $\Delta f_e/f_e$. Flow visualization corresponds to dimensionless modulation frequency $f_m/f_e = \frac{1}{4}$; $f_e/f_0^* = 0.95$; $Y_e/D = 0.267$.

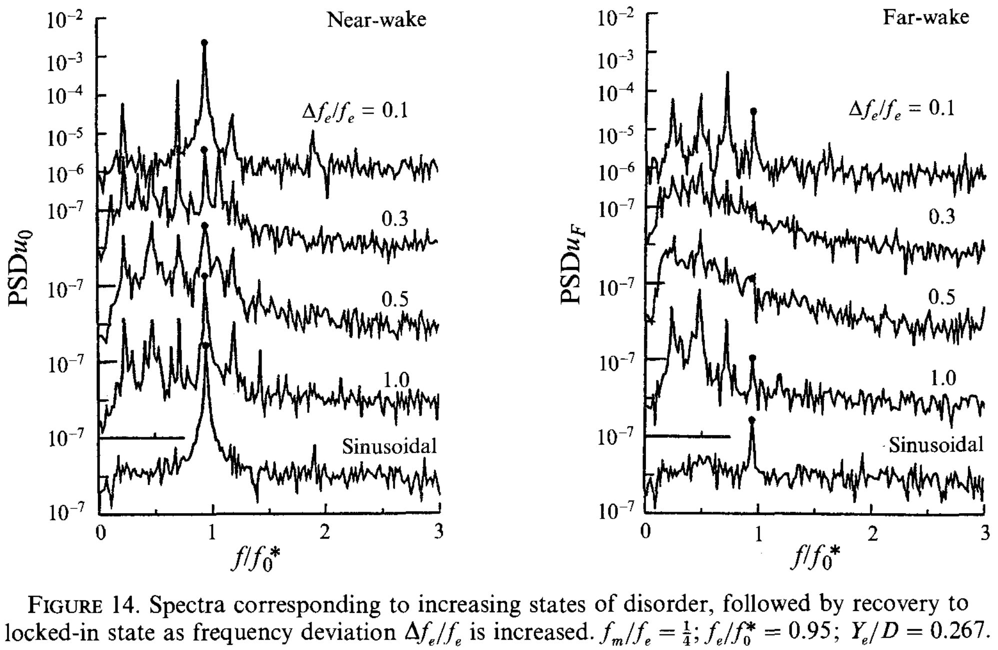


FIGURE 14. Spectra corresponding to increasing states of disorder, followed by recovery to locked-in state as frequency deviation $\Delta f_e/f_e$ is increased. $f_m/f_e = \frac{1}{4}$; $f_e/f_0^* = 0.95$; $Y_e/D = 0.267$.

excitation at $f_e/f_0^* = 0.95$. Most interesting, however, is that the onset of period-doubled response, i.e. $2f_m$-periodic response, is shifted to a lower value of modulation frequency $f_m/f_e = \frac{1}{4}$ in comparison with the occurrence of period doubling at $f_m/f_e = \frac{1}{3}$ for $f_e/f_0^* = 0.95$. After this period doubling has occurred, the level of the broadband fluctuations increases and the discrete spectral peaks are diminished, evident in the spectrum corresponding to $f_m/f_e = \frac{1}{5}$. This trend is in general accord with that occurring for $f_e/f_0^* = 0.95$ (figure 12*a*). In summary, it is evident that retuning the nominal excitation frequency to matched excitation postpones the onset of the first

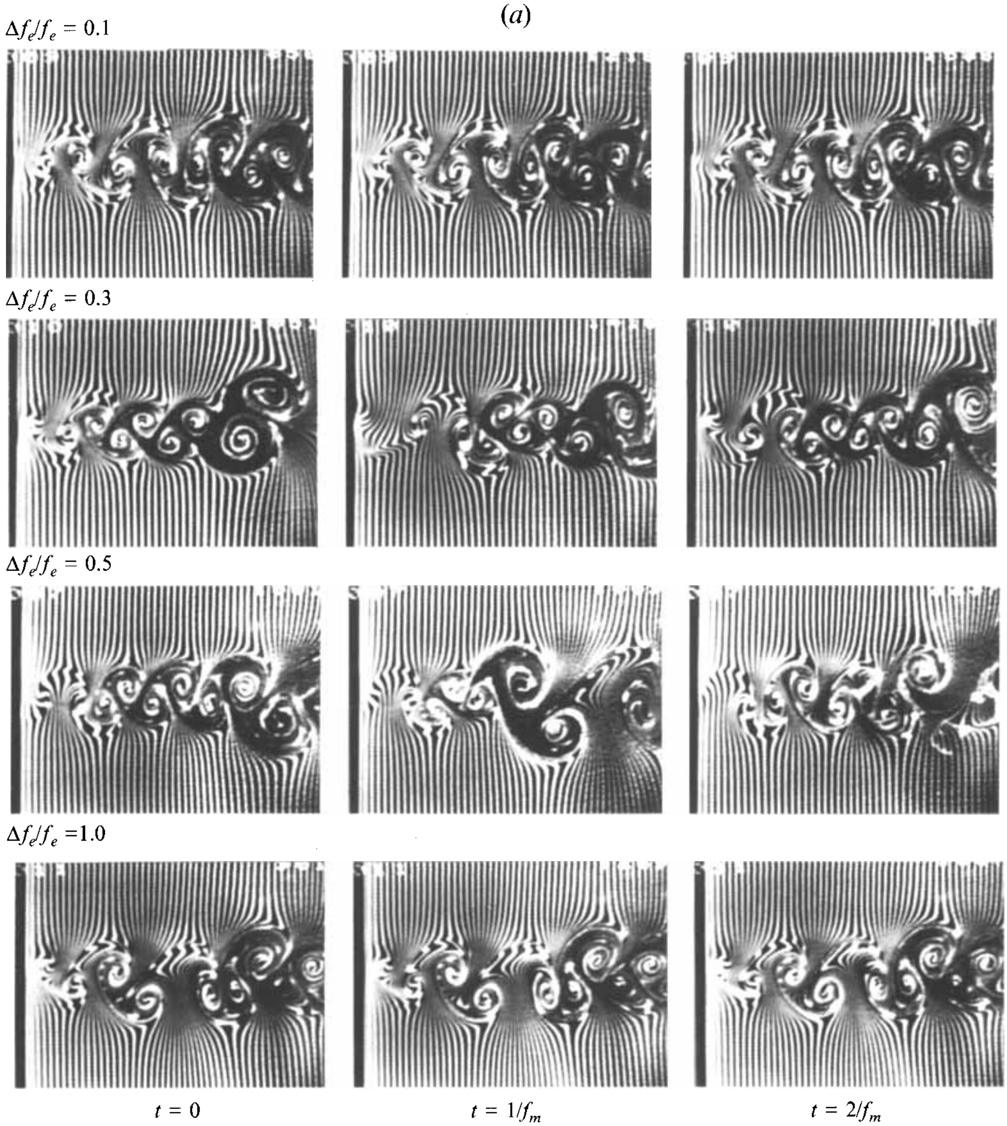


FIGURE 15(a). For caption see facing page.

stage of increasing disorder, i.e. $2f_m$-periodic response and thereby the onset of further stages of disorder. Correspondingly, the spectra in the far wake, shown in figure 12(d), exhibit more sharply tuned peaks, relative to the background, than their counterparts at $f_e/f_0^* = 0.95$.

Flow visualization (not shown) in downstream regions of the wake, extending to $x/D = 35$, for the nominal excitation frequency $f_e/f_0^* = 0.95$ shows a response of the wake structure as suggested by the spectra of figures 12(a) and 12(b): f_m-periodic at $f_m/f_e = \frac{1}{2}$; $2f_m$-periodic at $f_m/f_e = \frac{1}{3}$; and no detectable repetition at $f_m/f_e = \frac{1}{5}$.

5.2. *Destabilization by variation of frequency deviation*

An analogous type of transition from the f_m-periodic, locked-in response to a destabilized one can be attained by variation of the frequency deviation $\Delta f_e/f_e$ at a given value of modulation frequency f_m/f_e. The approach here is to lower the value of

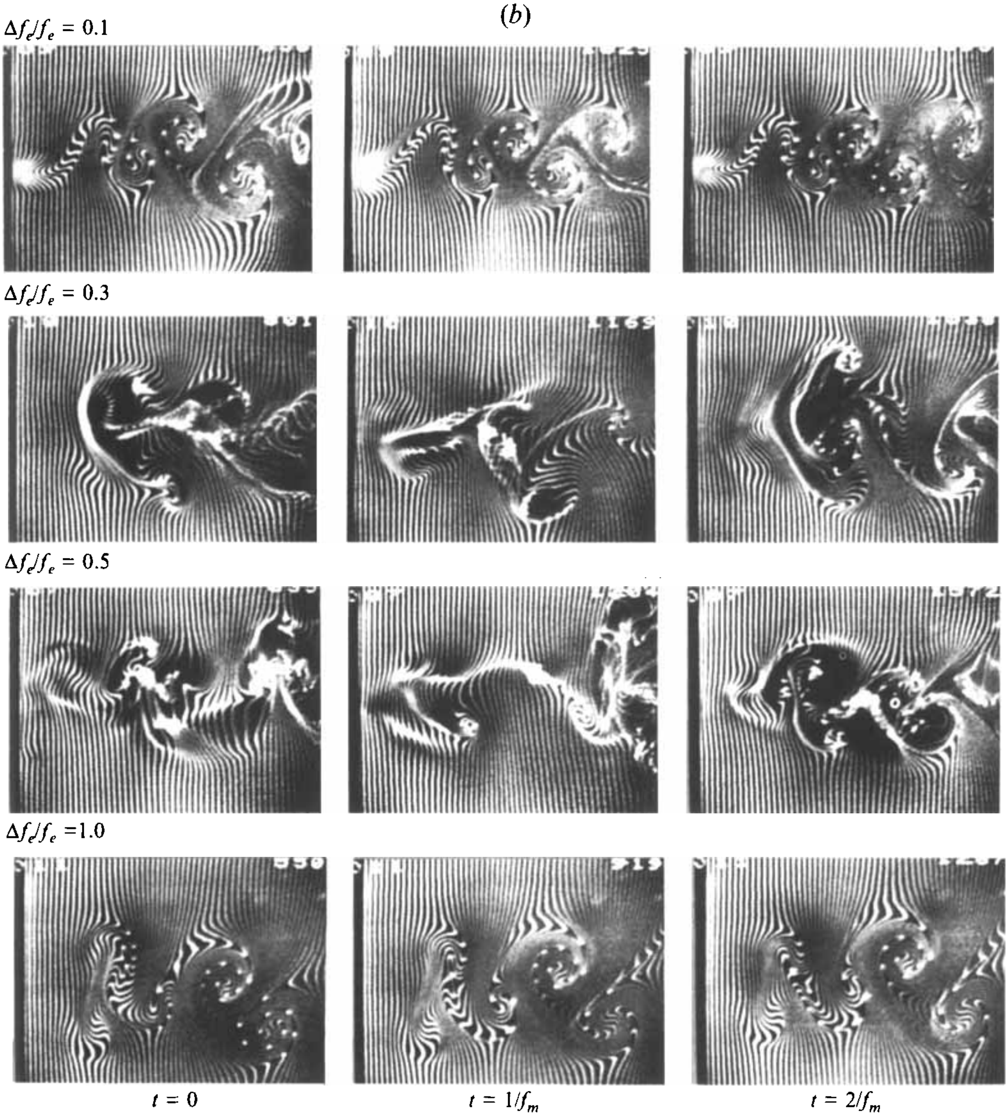


FIGURE 15. Illustration of flow structure (*a*) in near-wake region at $x/D = 5$, and (*b*) in far-wake region at $x/D = 35$, showing states of increasing disorder with increases in frequency deviation $\Delta f_e/f_e$, followed by recovery to highly organized wake at highest value of frequency deviation. $f_m/f_e = \frac{1}{4}$; $f_e/f_0^* = 0.95$; $Y_e/D = 0.267$.

modulation frequency f_m/f_e such that changes in $\Delta f_e/f_e$ drive the wake through a region of non-periodic f_m response. A schematic of the operating points is given in figure 13. The patterns of near-wake vortex formation for the f_m-periodic with f_e-locked-in response, the period-doubled response at $\frac{1}{2}f_m$, and non-coherent response of the wake are analogous to those described in previous sections, and therefore are not shown here.

The near-wake spectra of figure 14 show a typical locked-in response at the nominal excitation frequency f_e, presence of upper and lower sideband components and, in addition, a low-frequency component at the modulation frequency f_m. An increase in frequency deviation up to $\Delta f_e/f_e = 0.3$ produces a period-doubled response. At

$\Delta f_e/f_e = 0.5$, this period-doubled response gives way to broadening of the lower sidebands. At the highest-frequency deviation $\Delta f_e/f_e$, however, the degree of organization of the near wake is, to a large degree, recovered and the form of the spectrum is similar to the f_m-periodic, lock-in case at $\Delta f_e/f_e = 0.1$, except for presence of an additional sideband component.

In the downstream region of the wake, represented by the far-wake spectra on the right-hand side of figure 14, the f_m-periodic, lock-in response is essentially maintained at $\Delta f_e/f_e = 0.1$, while for higher values of frequency deviation $\Delta f_e/f_e = 0.3, 0.5$, rapid spectral broadening has occurred, with disappearance of a defined component at the nominal excitation frequency f_e. At the largest frequency deviation $\Delta f_e/f_e = 1.0$, the pronounced spectral peaks at the f_m sidebands are again apparent; they overshadow the component at excitation frequency f_e. The shapes of the spectra in the far wake therefore follow the near-wake response states defined in figure 13.

The visualizations of figures 15(*a*) and 15(*b*) show, respectively, the effect of $\Delta f_e/f_e$ at the values of $x/D = 5$ and 35. These photo layouts illustrate the mechanisms of the progressive destabilization, then restabilization, of the wake with increasing values of frequency deviation $\Delta f_e/f_e$. In figure 15(*a*), the wake at $\Delta f_e/f_e = 0.1$ is essentially phase-locked from one f_m cycle to the next, representing the f_m-periodic response. At $\Delta f_e/f_e = 0.3$, the vortex structure is very similar at $t = 0$ and $t = 2/f_m$, corresponding to the period-doubled, i.e. $2f_m$-periodic, response. At $\Delta f_e/f_e = 0.5$, the structure is not repetitive at $t = 1/f_m$ or $t = 2/f_m$. However, at the highest value of frequency deviation $\Delta f_e/f_e = 1.0$, the wake structure shows a remarkably repetitive, phase-locked character. These general observations are in accord with the spectra of figure 14.

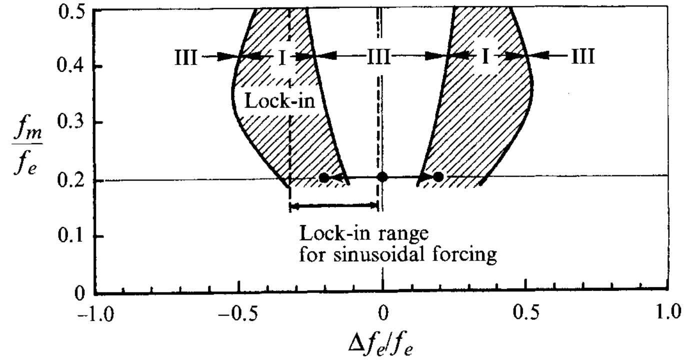
At the furthest downstream location, represented by the photos of figure 15(*b*) at $x/D = 35$, the structure is essentially phase-locked with period $1/f_m$ at $\Delta f_e/f_e = 0.1$. At $\Delta f_e/f_e = 0.3$, the flow structures shown in the left top portions of photos at $t = 0$ and $2/f_m$ suggest persistence of the period-doubled response, but there is no similarity in the right portions of these photos. At $\Delta f_e/f_e = 0.5$, there is no indication at all of periodicity. Finally, at $\Delta f_e/f_e = 1.0$, the phase-locked, i.e. f_m-periodic, response is recovered. These features over the range of $\Delta f_e/f_e$ are in accord with the corresponding spectra given in figure 14. At the extreme values of frequency deviation $\Delta f_e/f_e = 0.1$ and 1.0, the occurrence of the large-scale, phase-locked vortical structures is associated with the existence of well-defined lower f_m sideband components. The higher sideband components are completely filtered out by the streamwise evolution of the wake.

6. Restabilization of flow structure to lock-in state

When the bluff-body wake is subjected to purely sinusoidal excitation outside the region of lock-in, there are pronounced modulations of the flow structure. Karniadakis & Triantafyllou (1989*a*) demonstrated this concept of low-order chaotic behaviour in a numerical simulation. Lotfy & Rockwell (1993) have characterized these types of modulations experimentally. The question arises as to whether FM excitation can restabilize a normally aperiodic response of the wake to one which is f_m-periodic and f_e-locked-in. To illustrate this concept, we consider nominal excitation frequencies lying at the boundary of, and outside of, the lock-in envelope.

6.1. *Restabilization near the lock-in boundary*

The first case involves a nominal excitation frequency ratio $f_e/f_0^* = 1.089$, corresponding to the upper boundary of the lock-in range at an amplitude $Y_e/D = 0.267$, as illustrated in figure 2. In figure 16, which defines the response to f_m excitation, the



I ≡ Periodic at f_m; locked-in at f_e
III ≡ Non-Periodic at f_m; non-locked-in at f_e

FIGURE 16. Excitation condition (indicated by dots) for flow visualization representing restabilization of wake structure on plane of dimensionless modulation frequency f_m/f_e versus dimensionless frequency deviation $\Delta f_e/f_e$. $f_m/f_e = \frac{1}{5}$; $f_e/f_0^* = 1.089$; $Y_e/D = 0.267$.

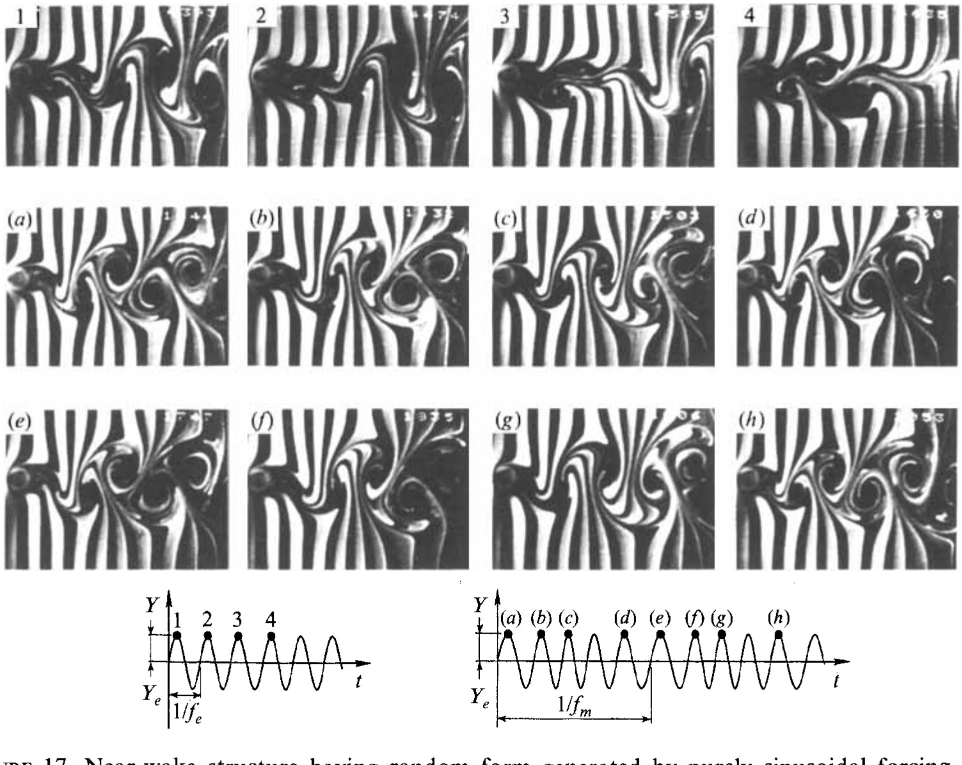


FIGURE 17. Near-wake structure having random form generated by purely sinusoidal forcing at $f_e/f_0^* = 1.089$ and restabilized, organized form generated by frequency-modulated (FM) excitation at $f_m/f_e = \frac{1}{5}$;$f_e/f_0^* = 1.089$; $\Delta f_e/f_e = 0.2$; $Y_e/D = 0.267$.

nominal excitation frequency is coincident with the line $\Delta f_e/f_e = 0$. The lock-in range for sinusoidal forcing is indicated by the bold horizontal line. For FM excitation, the hatched region defines the region of locked-in response. It is evident that a threshold value of frequency deviation $\Delta f_e/f_e = 0.13$ must be exceeded before lock-in can be

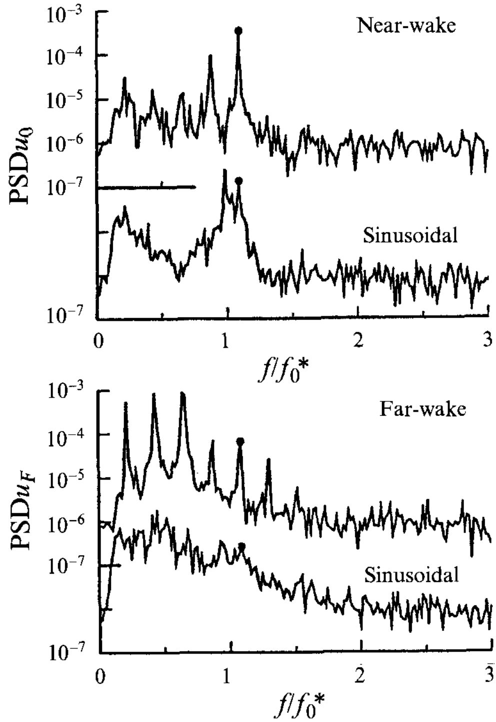


FIGURE 18. Spectra illustrating restabilization of near- and far-wake response due to FM excitation relative to sinusoidal excitation. $f_m/f_e = \frac{1}{5}$; $\Delta f_e/f_e = 0.2$; $f_e/f_0^* = 1.089$; $Y_e/D = 0.267$.

attained. Further, if $\Delta f_e/f_e$ is too large, $\Delta f_e/f_e \gtrsim 0.35$, then lock-in no longer exists. To illustrate the process of restabilization here, a value of $\Delta f_e/f_e = 0.2$ is selected; it is just above the threshold.

The flow visualization of figure 17 illustrates the wake response for purely sinusoidal and FM excitation. For purely sinusoidal excitation, the wake structure is non-periodic, evident from successive photos 1–4. Since, as will be shown subsequently, the spectrum for this aperiodic response involves a broadband concentration of energy centred at about $\frac{1}{5}f_e$, application of a modulation frequency $f_m/f_e = \frac{1}{5}$ was anticipated to be the most effective. The response of the wake at a relatively small frequency deviation $\Delta f_e/f_e = 0.2$ is illustrated in the bottom set of photos of figure 17. The wake structure is clearly f_m-periodic and locked-in, evident from comparison of pairs of photos *a*, *e*; *b*, *f*; and so on.

Spectra corresponding to the two cases of figure 17 are given in figure 18. In the near wake, for purely sinusoidal excitation, the peak at the excitation frequency f_e is relatively broadband, and there is broad concentration of energy centred about $\frac{1}{5}f_e$. The FM excitation produces resonant response at the nominal excitation frequency f_e and identifiable sidebands below the f_e component corresponding to $f_m/f_e = \frac{1}{5}$. In the downstream region of the wake, the spectrum corresponding to purely sinusoidal excitation is broadband, while that arising from the FM excitation shows sharp spectral peaks, with dominance of the lower sideband components. It is therefore possible to restabilize the wake over a very substantial streamwise extent by appropriate FM excitation.

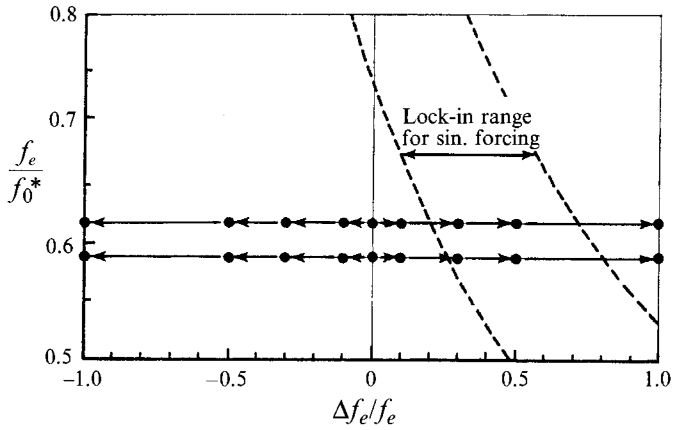


FIGURE 19. Excitation conditions (indicated as dots) for flow visualization corresponding to two values of nominal excitation frequency f_e/f_0^* well below inherent vortex formation frequency f_0^*. Dimensionless modulation frequency for flow visualization is $f_m/f_e = \frac{1}{4}$; $f_e/f_0^* = 0.589$ and 0.618; $Y_e/D = 0.267$.

6.2. *Restabilization near the inverse of golden mean*

When the nominal excitation frequency f_e/f_0^* is lowered to a value near or at the inverse of the golden mean, corresponding to the frequency ratio for the onset of chaos without phase-locking (Fein, Heutmaker & Gollub, 1985), the issue arises as to whether it is possible to influence the coherence of the wake response. In figure 19, the boundaries of the lock-in envelope generated by purely sinusoidal forcing are defined by the dashed lines. Excitation over a range of frequency deviation $\Delta f_e/f_e$ is addressed here for two values of nominal frequency: $f_e/f_0^* = 0.618$ and 0.589.

Figure 20 shows that, in the near wake, at $f_e/f_0^* = 0.589$ (*a*, *b*), purely sinusoidal excitation at f_e produces large-amplitude peaks at the inherent vortex formation frequency f_0^* and at the difference frequency (beat frequency) $f_0^* - f_e$; it is not possible to elicit a response at the nominal excitation frequency, which is indicated by the dot. This same spectral distribution is maintained at a low value of frequency deviation $\Delta f_e/f_e = 0.1$. At a higher value of frequency deviation $\Delta f_e/f_e = 0.3$, both the lower and upper f_m sideband components start to become apparent, although several of them are well defined. At $\Delta f_e/f_e = 0.5$, all three of the lower and upper f_m sidebands are sharply defined in a sense characteristic of f_m-periodic response. When the frequency deviation $\Delta f_e/f_e$ is extended to 1.0, the general form of the response is similar to that at $\Delta f_e/f_e = 0.5$, except for loss of one of the upper f_m sidebands.

This attainment of a substantial spectral component at the excitation frequency $f_e = 0.589 f_0^*$, together with resonant sidebands protruding well above the background level, is due to satisfaction of two conditions. The first condition is a sufficiently large value of frequency deviation such that the instantaneous frequency f_e extends into the lock-in region for purely sinusoidal excitation; this is illustrated in figure 19. The second condition is coincidence of the inherent vortex formation frequency with an upper sideband of the FM excitation. In the present situation, it corresponds closely to the third upper sideband at $f/f_0^* = f_e/f_0^* + \frac{3}{4}(f_e/f_0^*) = 0.589 + 0.442 = 1.03$. In fact, based on experiments at other values of nominal excitation frequency below $f_e/f_0^* = 1.0$, the general statement can be made that whenever one of the f_m sidebands is coincident with the inherent instability frequency f_0^*, a resonant, f_m-periodic response of the near-wake structure is produced. In other words, the versatility of FM excitation

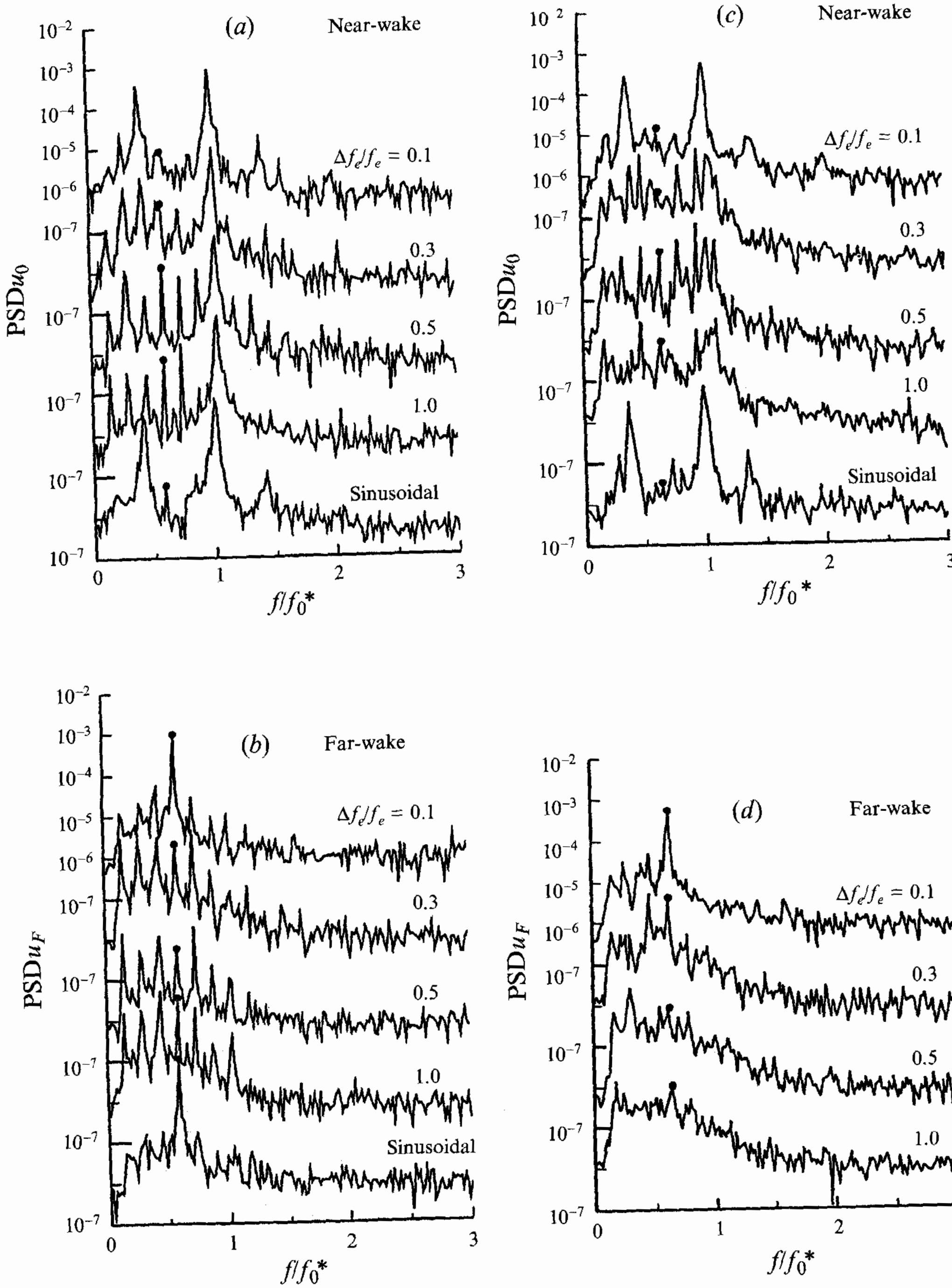


FIGURE 20. Spectra corresponding to case of restabilized wake response at $f_e/f_0^* = 0.589$ (*a, b*) and non-restabilized wake response at $f_e/f_0^* = 0.618$ (*c, d*) in near- and far-wake regions. $f_m/f_e = \frac{1}{4}$; $Y_e/D = 0.267$.

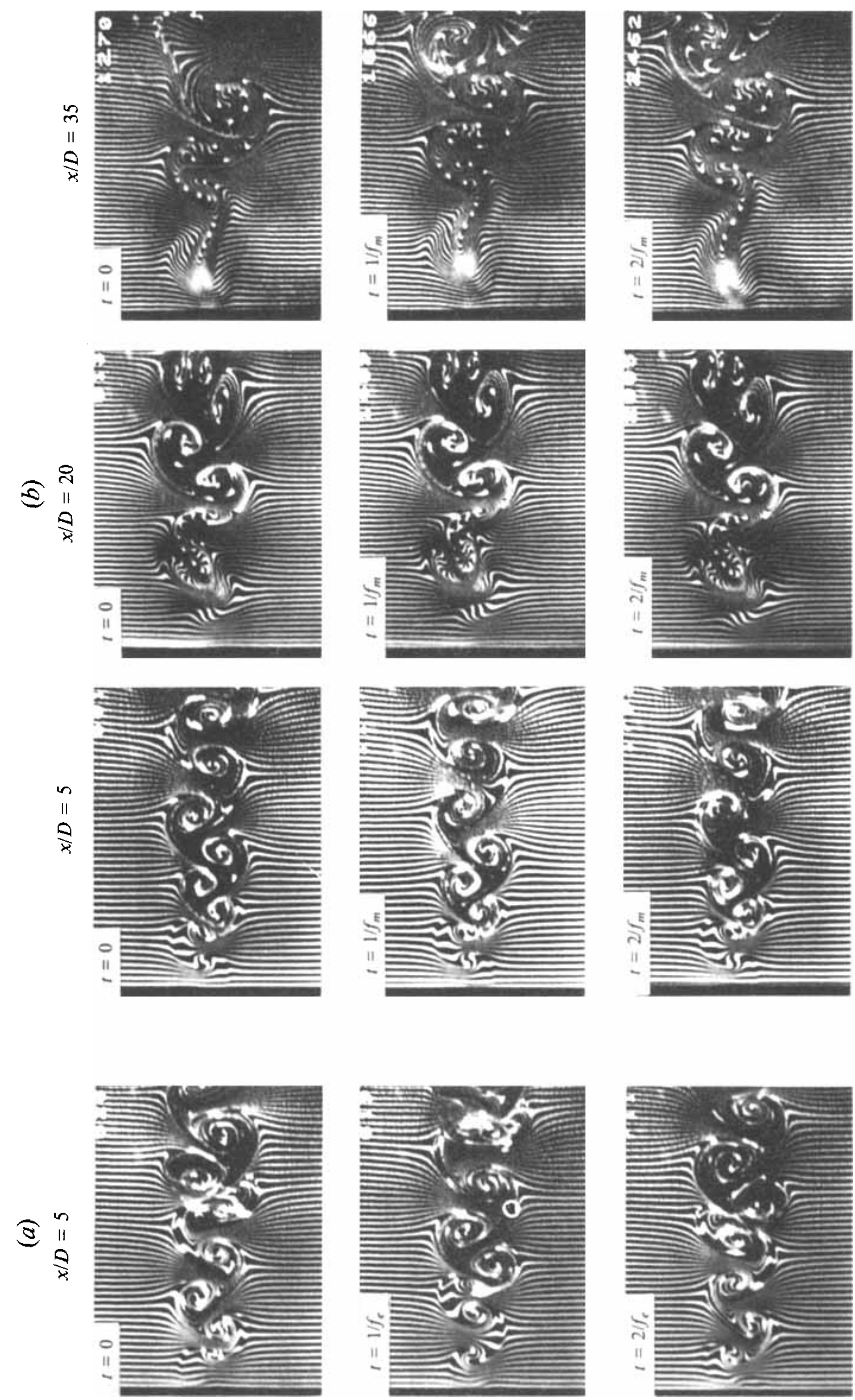


FIGURE 21. Visualization of non-periodic and non-locked-in response (*a*) at excitation frequency f_e due to purely sinusoidal forcing, in comparison with response (*b*) which is periodic at modulation frequency f_m and non-locked-in at excitation frequency f_e due to FM excitation. $f_m/f_e = \frac{1}{4}$; $f_e/f_0^* = 0.589$; $\Delta f_e/f_e = 0.5$; $Y_e/D = 0.267$.

is that a so-called resonant spectral response does not require coincidence of the excitation frequency f_e with f_0^*, but simply coincidence of one of the FM sidebands.

The corresponding spectra in the far wake are illustrated at the lower left of figure 20(*b*). For sinusoidal excitation and FM excitation at $\Delta f_e/f_e = 0.1$, there is a well-defined resonant response at the forcing frequency indicated by the dot. This predominant peak is because the forcing frequency $f_e/f_0^* = 0.598$ is close to the subharmonic $f_e/f_0^* = 0.589$; in the naturally evolving (unforced) wake, the subharmonic dominates in the downstream region due to the increase in lengthscale of the locally unstable flow (Cimbala, Nagib & Roshko 1988). At higher values of $\Delta f_e/f_e = 0.3$, 0.5, and 1.0, the sharp spectral peaks at the excitation component and its lower and first upper sidebands dominate the spectra; the component at the inherent instability frequency f_0^* no longer plays a significant role. In contrast to the resonant response shown in figure 20(*a*, *b*) for $f_e/f_0^* = 0.589$, the spectra shown in figure 20(*a*, *d*) show that, at the inverse of the golden mean $f_e/f_0^* = 0.618$, it is not possible to elicit a resonant f_m-periodic response, irrespective of the value of frequency deviation. A contributing factor is that an upper sideband of the FM excitation is not sufficiently close to the natural frequency f_0^*. It is, however, possible to induce a high level of background fluctuations at low frequency, especially evident in the far-wake spectra of figure 20(*d*).

Representative flow visualization in the downstream region of the wake is given in figure 21, corresponding to purely sinusoidal excitation and a frequency deviation $\Delta f_e/f_e = 0.5$. The case of sinusoidal excitation is represented in figure 21(*a*); the wake structure shows no apparent periodicity. For FM excitation at a frequency deviation $\Delta f_e/f_e = 0.5$, represented by figure 21(*b*), periodicity is attained at frequency f_m. This f_m-periodic response extends over the entire streamwise extent of the wake.

7. Concluding remarks

Purely sinusoidal excitation of a cylinder generates a region of locked-in response of the wake provided the excitation frequency is sufficiently close to the vortex formation frequency. Within this region of lock-in, the pattern of vortex formation is highly repetitive from cycle to cycle of the cylinder motion; moreover, the vortices are shed at approximately the same phase of the cylinder motion from one cycle to the next. Outside this region of lock-in, irregular patterns of vortices are generated, and the timing of their formation relative to the cylinder motion is not repetitive.

Proper application of frequency-modulated (FM) excitation can substantially modify the response of near- and far-wake regions, relative to what occurs for purely sinusoidal excitation. Among these modifications are: an expansion of the extent of the lock-in range of vortex formation, based on instantaneous deviations from the nominal excitation frequency; destabilization of the organized wake at a nominal excitation frequency for which lock-in normally occurs during purely sinusoidal excitation; and restabilization of the wake from a relatively unorganized state generated by sinusoidal excitation. The principal control parameters of frequency-modulated excitation are the nominal value of the excitation frequency relative to the inherent vortex formation frequency, the frequency deviation from the nominal excitation frequency, and the modulation frequency. Substantial control can be obtained by variation of any one, or a combination, of these parameters.

The concept of locked-in response of the wake is well known for sinusoidal excitation. Outside this locked-in region, the highly repetitive vortex formation from cycle to cycle of the cylinder motion gives way to quasi-ordered or random variations of the near-wake structure. During application of the frequency-modulated excitation, there are instantaneous excursions, or frequency deviations, from the mean value of excitation frequency. In concept, the potential exists for obtaining a locked-in response of the wake over the entire range of instantaneous frequency deviation. In fact, by tuning the modulation frequency to a sufficiently high value, it is possible to attain a locked-in response for instantaneous frequency deviations as large as the nominal excitation frequency.

Destabilization of the vortex pattern in the near wake, and consequently the far wake as well, can be attained at nominal excitation frequencies lying within the envelope of locked-in response to purely sinusoidal excitation. By keeping the frequency deviation constant and successively lowering the modulation frequency, it is possible to generate successive states of increasing disorder of the wake. The first state, which is highly ordered, involves locked-in response at the nominal excitation frequency and simultaneously periodic response at the modulation frequency. The second state is a period-doubled version of the first one; it is characterized by a wake structure that is periodic at half the modulation frequency. The third- and higher-order states are defined by complete loss of periodicity at the modulation frequency or one of its subharmonics. It should be emphasized, however, that the video records were not long enough to ascertain whether this last state exhibited a very long-term periodicity at, for example, a frequency one-fourth or one-eighth the modulation frequency. A particularly intriguing aspect of this concept of destabilization is the high sensitivity to the nominal value of excitation frequency. When the nominal frequency is matched to the inherent vortex formation frequency from the cylinder, the onset of the period-doubled response is delayed to a lower value of modulating frequency.

An equivalent form of destabilization can be attained by varying the frequency deviation while keeping the modulation frequency constant. In this case, it is possible to witness an analogous sequence of states of increasing disorder. At high values of frequency deviation, however, the wake may actually recover to a locked-in response after undergoing a destabilized response at a lower value of frequency deviation.

Restabilization of the wake to an organized, locked-in structure is attainable when it exhibits disorganized or random patterns of vortex formation under sinusoidal excitation. The case selected for restabilization corresponds to a nominal excitation frequency at the upper boundary of the lock-in region of sinusoidal excitation. The random response of the wake can be driven to a structure that is locked-in at the excitation frequency and periodic at the modulation frequency. Attempts to restabilize the wake pattern at a frequency near or at that of the inverse of the golden mean frequency reveal two conditions that must be satisfied to attain a response that is non-locked-in at the excitation frequency and periodic at the modulation frequency. First, the instantaneous frequency deviation must attain a sufficiently high value to lie within the steady-state, locked-in region corresponding to sinusoidal excitation. Secondly, one of the sidebands of the frequency-modulated excitation must be coincident, or nearly so, with the inherent vortex formation frequency from the stationary cylinder.

For all of the foregoing types of frequency-modulated control, the character of the far wake is intimately related to the near-wake structure induced by the FM excitation. Depending on the type of vortex pattern induced in the near wake, highly organized or broadband spectra of the velocity field can be generated in the far-wake region. A general tendency, irrespective of the particulars of the response spectra, is for the upper FM sidebands to be filtered out as the wake evolves in the downstream direction. This filtering is a natural consequence of the increasing value of transverse lengthscale of the wake with increasing streamwise distance. The band of unstable frequencies that can be supported at larger streamwise distances shifts to successively lower values, thereby providing a type of bandpass filter with a successively lower centre frequency. As a consequence, only the lower range of sidebands survives the streamwise evolution of the wake. This concept is related to that advanced by Cimbala *et al.* (1988) for the naturally evolving wake in absence of forced excitation. In their study, the emergence of a subharmonically dominated flow structure in the downstream wake was attributed to the lower value of local instability frequency of the downstream wake. One must exercise caution, however, in relating the mechanisms of spectral energy transfer in the study of Cimbala *et al.* (1988) to those of the present investigation. The highly nonlinear excitation of the cylinder provides an inherently different set of initial conditions for the streamwise evolution of the wake.

From a practical standpoint, it is evident that the types of wake response due to forced FM excitation may be exploited to advantage in the areas of mixing, heat transfer, and combustion. It would be worthwhile to characterize the unsteady and mean concentration fields, heat transfer coefficients and burning characteristics in relation to the classes of near-wake structure generated in this study. In the area of flow-induced vibration and noise generation, it is the corresponding self-excited response of the elastically mounted cylinder that is of eventual interest. In the field, the self-excited response of cables and structures is known to exhibit frequency modulations. The link between these self-excited modulations and those due to controlled forcing should be pursued.

The strong connection between the near- and far-wake regions observed in this study is analogous to that observed by Williamson & Prasad (1993) for the three-dimensional structure of the wake from a stationary cylinder. By controlling the occurrence of either parallel or oblique shedding in the near-wake region, the direct consequence on the structure of the far wake could be observed. This suggests a fascinating prospect for further research: the survival of three-dimensional, frequency-modulated imprints into the far-wake region.

The authors are pleased to acknowledge the support of the National Science Foundation and the Office of Naval Research during the course of this investigation.

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